

INDIAN SCHOOL MUSCAT

CLASS 12

DEPARTMENT OF MATHEMATICS

W.S (3)

MATRICES AND DETERMINANTS

- 1) Construct a 3x3 matrix $A = [a_{ij}]$ if, a) $a_{ij} = \frac{|2i-3j|}{2}$
- 2) Find a, b, c, d if $\begin{bmatrix} 2a+b & a-2b \\ 5c-d & 4c+3d \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 11 & 24 \end{bmatrix}$
- 3) Find X such that $X \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$
- 4) $A = \begin{bmatrix} 0 & -\tan \frac{\alpha}{2} \\ \tan \frac{\alpha}{2} & 0 \end{bmatrix}$ Show that $I + A = (I - A) \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$
- 5) $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$. using PMI, Prove that $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$ for all $n \in \mathbb{N}$
- 6) Prove that Every square matrix can be uniquely expressed as the sum of a symmetric matrix and a skew symmetric matrix.
- 7) Express the following matrices as the sum of a symmetric matrix and a skew symmetric matrix. a) $A = \begin{bmatrix} 3 & 2 & 3 \\ 4 & 5 & 3 \\ 2 & 4 & 5 \end{bmatrix}$
- 8) $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$ find $A^2 - 5A + 6I$.
- 9) Compute $\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix}$
- 10) Obtain the inverse of the matrix using elementary transformations:-
 a) $\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$
- 11) Prove the following without expanding:

$$a) \begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$$

$$b) \begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3$$

$$c) \begin{vmatrix} a^2+1 & ab & ca \\ ab & b^2+1 & bc \\ ca & bc & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$$